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Two-Loop $\mathcal{O}(\alpha_s^2)$ Correction to the $H \to b\bar{b}$ Decay Rate Induced by the Top Quark

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Abstract

We present the two-loop QED and QCD corrections to the $f\bar{f}H$ Yukawa coupling that are induced by the exchange of a virtual photon or gluon, respectively, with a heavy-fermion loop inserted. As an application, we study the corresponding $\mathcal{O}(\alpha_s^2 M_H^2/m_t^2)$ correction to the $H \to b\bar{b}$ decay rate.

A Higgs boson with $M_H \lesssim 135$ GeV decays dominantly to $b\bar{b}$ pairs [1]. This decay mode will be of prime importance for Higgs-boson searches at LEP 2 [2], the Tevatron [3]—or a possible 4-TeV upgrade thereof [4]—, and the next e^+e^- linear collider [5]. Techniques for the measurement of the $H \to b\bar{b}$ branching fraction at a $\sqrt{s} = 500$ GeV e^+e^- linear collider have been elaborated in Ref. [6].

The present knowledge of quantum corrections to the $H \to b\bar{b}$ decay rate has been reviewed very recently in Ref. [1]. The full one-loop electroweak corrections to this process are well established [7, 8]. At two loops, the universal [9] and non-universal [10] $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections have recently been calculated. The pure QCD corrections are most significant numerically. In $\mathcal{O}(\alpha_s)$, their full m_b dependence is known [11]. In $\mathcal{O}(\alpha_s^2)$, the first [12] and second [13] terms of the expansion in m_b^2/M_H^2 have been found.

The results of Refs. [12, 13] take into account five quark flavours. In $\mathcal{O}(\alpha_s^2)$, there are additional contributions involving a virtual top quark, which are not formally suppressed. These corrections may be viewed as the absorptive parts of the three-loop Higgs-boson self-energy diagrams that involve a top-quark loop, a bottom-quark loop, and two gluon lines. These diagrams may be divided in two classes: the so-called double-triangle diagrams and the the diagrams where a gluon line with a top-quark loop insertion is attached in all possible ways to the one-loop seed diagram involving the bottom quark. The first class has been considered just recently [14]. In this letter, we shall study the second class.

We shall proceed along the lines of Ref. [15],[1] where the two-loop $f\bar{f}\gamma$ vertex correction due to a virtual massive fermion, F, was derived in QED, assuming $m_f^2 \ll |s|$, where s

[1] We take this opportunity to correct two misprints in the published version of Ref. [15], which are absent in the preprint. In the second line of Eq. (5), the terms $-\varphi^2$ and $53/2$ should be replaced by $+\varphi^2$ and $53/3$.

is the photon invariant mass squared. In this limit, only the Dirac Form factor, which multiplies γ^μ, survives. It has the form [15]

$$\Gamma^\mu = \gamma^\mu \left[1 + \frac{\alpha}{\pi} Q_f^2 F_1(s) + \left(\frac{\alpha}{\pi}\right)^2 Q_f^2 N_F Q_F^2 F_2^{(F)}(s) + \cdots\right], \tag{1}$$

where Q_f is the electric charge of f (in units of the positron charge), $N_f = 1$ (3) for leptons (quarks), and the dots represent other contributions in $\mathcal{O}(\alpha^2)$ and higher orders. By adjusting coupling constants and colour factors, one immediately obtains the corresponding QCD expansion for the case when f and F are quarks. Specifically, one substitutes $\alpha \to \alpha_s$, $Q_f^2 \to C_F = (N_c^2 - 1)/(2N_c) = 4/3$, and $Q_F^2 \to T = 1/2$, where $N_c = 3$ is number of colours and $\operatorname{tr} t^a t^b = T\delta^{ab}$, with t^a $(a = 1, \ldots, 8)$ being the generators of the quark (defining) representation of $\mathrm{SU}(N_c)$. In particular, the m_t-dependent $\mathcal{O}(\alpha_s^2)$ correction to the non-singlet contribution to $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ is given by

$$\delta R_{NS} = 2\left(\frac{\alpha_s}{\pi}\right)^2 C_F T \operatorname{Re} F_2^{(F)}(s), \tag{2}$$

which is in agreement with Ref. [16].

Similarly to Eq. (1), the QED expansion of the $f\bar{f}H$ coupling may be written as

$$\Gamma = 1 + \frac{\alpha}{\pi} Q_f^2 H_1(s) + \left(\frac{\alpha}{\pi}\right)^2 Q_f^2 N_F Q_F^2 H_2^{(F)}(s) + \cdots. \tag{3}$$

We have $H_1(s) = (1/4)\hat{\delta}_{em}|_{h=s}$, where $\hat{\delta}_{em}$ may be found in Eq. (2.18) of Ref. [8]. $H_1(s)$ is plagued by an infrared singularity, which is cancelled by a similar contribution from soft-photon bremsstrahlung when a physical observable, such as the $H \to f\bar{f}$ decay rate, is computed. In Ref. [8], the infrared singularity is regularized by an infinitesimal photon mass, μ. Up to terms proportional to m_f^2/s, one has

$$H_1(s) = \frac{L}{2}(z-1) - \frac{z^2}{4} + \frac{7}{2}\zeta(2) - \frac{1}{2}, \tag{4}$$

where $\zeta(2) = \pi^2/6$, $L = \ln(\mu^2/m_f^2)$, and $z = \ln(-s/m_f^2 - i\epsilon)$. In the following, we shall need an expression for $H_1(s)$ appropriate for a neutral gauge boson with arbitrary mass, $\sqrt{\sigma}$. From Eq. (2.6) of Ref. [8] one extracts

$$H_1(s,\sigma) = \frac{1}{2}\left(\mathrm{Li}_2\left(1 + \frac{s + i\epsilon}{\sigma}\right) - \zeta(2)\right), \tag{5}$$

where Li_2 denotes the dilogarithm.

The Feynman diagrams contributing to $H_2^{(F)}(s)$ are depicted in Fig. 1. $H_2^{(F)}(s)$ is infrared-finite and devoid of mass singularities associated with f. It may be conveniently calculated by convoluting $H_1(s,\sigma)$ with the imaginary part of the one-loop contribution of F to the photon self-energy,

$$\operatorname{Im}\Pi_{AA}^{(F)}(s) = \frac{\alpha}{3} N_F Q_F^2 s P(s), \tag{6}$$

where

$$P(s) = \left(1 + \frac{2m_F^2}{s}\right)\sqrt{1 - \frac{4m_F^2}{s}}. \tag{7}$$

The precise relation reads

$$H_2^{(F)}(s) = \frac{1}{3}\int_{4m_F^2}^{\infty} \frac{d\sigma}{\sigma}\, P(\sigma) H_1(s,\sigma). \tag{8}$$

After a straightforward calculation, one finds

$$\begin{aligned}
3H_2^{(F)}(s) &= \mathrm{Li}_3(-\rho_-^2) + \frac{1}{3}\left(5 - \frac{1}{r}\right)\sqrt{1 + \frac{1}{r}}\left(\mathrm{Li}_2(-\rho_-^2) + \varphi^2 + \frac{\zeta(2)}{2}\right) - \frac{2}{3}\varphi^3 - \zeta(2)\varphi \\
&\quad + \frac{2}{3}\left(-\frac{14}{3} + \frac{1}{r}\right)\gamma - \zeta(3) + \frac{82}{27} - \frac{2}{3r}, \qquad r \le -1, \\
&= \mathrm{Cl}_3(2\Phi) - \frac{1}{3}\left(5 - \frac{1}{r}\right)\sqrt{-\frac{1}{r} - 1}\,\mathrm{Cl}_2(2\Phi) \\
&\quad + \frac{2}{3}\left(-\frac{14}{3} + \frac{1}{r}\right)\gamma - \zeta(3) + \frac{82}{27} - \frac{2}{3r}, \qquad -1 \le r \le 0, \\
&= \mathrm{Li}_3(r_-^2) + \frac{1}{3}\left(5 - \frac{1}{r}\right)\sqrt{1 + \frac{1}{r}}\left(\mathrm{Li}_2(r_-^2) + f^2 - \zeta(2)\right) - \frac{2}{3}f^3 + 2\zeta(2)f \\
&\quad + \frac{2}{3}\left(-\frac{14}{3} + \frac{1}{r}\right)g - \zeta(3) + \frac{82}{27} - \frac{2}{3r} \\
&\quad + i\pi\left[f^2 - \frac{1}{3}\left(5 - \frac{1}{r}\right)\sqrt{1 + \frac{1}{r}}f + \frac{14}{9} - \frac{1}{3r}\right], \qquad r \ge 0,
\end{aligned} \tag{9}$$

where $\zeta(3) = 1.20205\,69031\,59594\,28540\ldots$, Li_3 is the trilogarithm, Cl_2 (Cl_3) is the (generalized) Clausen function of second (third) order,

$$\begin{aligned}
&r = \frac{s}{4m_F^2}, \qquad \rho_\pm = \sqrt{-r} \pm \sqrt{-r-1}, \qquad r_\pm = \sqrt{1+r} \pm \sqrt{r}, \\
&\varphi = \ln\rho_+ = \operatorname{arcosh}\sqrt{-r}, \qquad \Phi = \arcsin\sqrt{-r}, \qquad f = \ln r_+ = \operatorname{arsinh}\sqrt{r}, \\
&\gamma = \ln(\rho_+ + \rho_-) = \ln\left(2\sqrt{-r}\right), \qquad g = \ln(r_+ - r_-) = \ln\left(2\sqrt{r}\right).
\end{aligned} \tag{10}$$

Note that $H_1(s)$ and $H_2^{(F)}(s)$ develop imaginary parts above the $f\bar{f}$-pair production threshold, i.e., for $s > 4m_f^2 = 0$.

It is instructive to study the limiting behaviour of $H_2^{(F)}(s)$. For $s \to -\infty$ and $s \to -0$, one has

$$\begin{aligned}
3H_2^{(F)}(s) &= -\frac{2}{3}\gamma^3 + \frac{5}{3}\gamma^2 - \left(\zeta(2) + \frac{28}{9}\right)\gamma - \zeta(3) + \frac{5}{6}\zeta(2) + \frac{82}{27} + \frac{3\gamma}{2r} \\
&\quad + \mathcal{O}\left(\frac{\gamma^2}{r^2}\right), \qquad r \ll -1, \\
&= \frac{r}{5}\left(-4\gamma + \frac{107}{15}\right) + \frac{r^2}{35}\left(6\gamma - \frac{529}{70}\right) + \mathcal{O}(r^3\gamma), \qquad -1 \ll r \le 0,
\end{aligned} \tag{11}$$

respectively. The corresponding expansions for positive s may be found by analytic continuation, i.e., by substituting $\gamma = g - i\pi/2$. In compliance with the Appelquist-Carazzone theorem [17], the loop fermion, F, decouples for $m_F^2 \gg |s|$.

In Fig. 2, $\mathrm{Re}\, H_2^{(F)}(s)$ is plotted as a function of $r = s/(4m_F^2)$. At $r \approx 5.62$, $\mathrm{Re}\, H_2^{(F)}(s)$ assumes its maximum value, 0.432. Its expansions, which emerge from Eq. (11) through analytic continuation, are also shown. Obviously, they provide an excellent approximation for $r \gtrsim 1$ and $r \lesssim 1$, respectively.

The QCD expansion of the $f\bar{f}H$ coupling for the case when f and F are quarks may be obtained from Eq. (3) through the substitutions specified below Eq. (1). As an application, we consider the m_t-dependent $\mathcal{O}(\alpha_s^2)$ correction to the $H \to b\bar{b}$ decay rate arising from the Feynman diagrams in Fig. 1 with $f = b$, $F = t$, and the photon replaced by a gluon. Similarly to Eq. (2), the relative shift is

$$\begin{aligned}\frac{\delta\Gamma\left(H \to b\bar{b}\right)}{\Gamma\left(H \to b\bar{b}\right)} &= 2\left(\frac{\alpha_s}{\pi}\right)^2 C_F T\, \mathrm{Re}\, H_2^{(t)}(M_H^2) \\ &= \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{M_H^2}{45m_t^2}\left(2\ln\frac{m_t^2}{M_H^2} + \frac{107}{15}\right) + \mathcal{O}\left(\frac{M_H^4}{m_t^4}\ln\frac{m_t^2}{M_H^2}\right)\right], \qquad (12)\end{aligned}$$

where the second line is appropriate for $M_H \lesssim 2M_W$, where the $H \to b\bar{b}$ decay rate is most relevant phenomenologically. The coefficient of $(\alpha_s/\pi)^2$ in Eq. (12) ranges between 2.38×10^{-2} for $(M_H, m_t) = (60, 200)$ GeV and 0.502 for $(M_H, m_t) = (1000, 150)$ GeV; its value at $M_H = 2m_t$ is 0.353. This has to be compared with the value 29.14671 due to five massless quark flavours [12]. On the other hand, the finite-m_b term in $\mathcal{O}(\alpha_s^2)$ has the coefficient $-87.72459\,(\overline{m}_b(M_H)/M_H)^2$ [13], where $\overline{m}_b(\mu)$ is the bottom-quark $\overline{\mathrm{MS}}$ mass at renormalization scale μ. Assuming $\alpha_s(M_Z) = 0.118$ [18] and $m_b = 4.72$ GeV [19], this amounts to -0.200 (-2.37×10^{-2}) at $M_H = 60$ GeV ($2M_W$). In conclusion, the $\mathcal{O}(\alpha_s^2 M_H^2/m_t^2)$ correction to $\Gamma\left(H \to b\bar{b}\right)$ arising from the Feynman diagrams shown in Fig. 1 is comparable in size with the $\mathcal{O}(\alpha_s^2 m_b^2/M_H^2)$ correction, but has the opposite sign.

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FIGURE CAPTIONS

Figure 1: Feynman diagrams pertinent to the two-loop $f\bar{f}H$ vertex correction induced by a virtual heavy fermion, F.

Figure 2: $\mathrm{Re}\,H_2^{(F)}(s)$ as a function of $r = s/(4m_F^2)$ [see Eq. (9)] and its expansions [see Eq. (11)].